Geometry Round One Solutions

1. The surface area is comprised of 2 octagons and 8 rectangles. The formula for an area of an octagon is where s is the side length of the octagon. Plugging in 4 for s gives . We multiply this by 2 to account for both octagons in the prism to get . Then, we add the areas of the 8 4x6 rectangles which total to an area of 192. Adding 192 and gives , **D.**
2. The volume of a hexagonal prism is the area of the base times the height. The area of a hexagon is , plugging in 8 for s, the area of the base is . Multiplying this by 8 (the height) gives the volume of the prism to be . The pyramid on top has a volume equal to the base times the height divided by 3. The area of the hexagon is and the height is 4. . Adding the volumes of the prism and the pyramid give . The question asks for the volume of two yurts, so the answer is , **D.**
3. The sum of the angles of a triangle equals 180°. Adding up all of the angles of the triangle gives , so . Plugging in 17 for all the angles gives the angles of the triangles as °, and The negative difference between the least and greatest angles is , **B.**
4. The hypotenuse of the triangle is . We can use the Pythagorean theorem to find the other leg of the triangle. . The length of the altitude to the hypotenuse of a right triangle can be found my multiplying the legs and dividing by the hypotenuse. **A.**
5. The formula for the surface area of a sphere is , where is the radius. The diameter is so the radius is . Plugging in for , **B.**
6. The supplement of Angle R is The complement of is . Setting gives . Plugging in 14 into angle R, , **C.**
7. Euler’s line is a straight line that goes through the circumcenter, orthocenter, and centroid of a circle. The incenter is not always on Euler’s line, **C.**
8. The radius of an incircle of a triangle is found by taking the area of the triangle and dividing the semi perimeter of the triangle. The area of the triangle can be found by using Heron’s formula, , where is the semi perimeter and a, b, and c are the sides. The semi perimeter is 18/2=9. Plugging in everything we get the area of the triangle as . The radius of the incircle is . The area of the incircle is , **B.**
9. The centroid is the average of the coordinates of the three vertices of the triangle. and . The centroid is **C.**
10. The triangle being rotated about the y-axis will form a cone with radius 2 and height 8. The volume of a cone is . Plugging in r and h, the volume of the cone is . This volume is equal to the volume of a sphere. The volume of a sphere is . Therefore, r = 2, so the diameter = 4, **B.**
11. The shortest side of a triangle is opposite the smallest angle of a triangle. In a 30-60-90 triangle, the side opposite of the 60° angle is the length of the smallest side multiplied by . The smallest side is , so the other leg of the triangles is 9. The area of the triangle is , **C.**
12. An apothem of a hexagon is the line segment from the middle of a side of the hexagon to the center of the hexagon. The area of a hexagon is which in this problem equals , so s = 8. If the side of the equilateral triangle is 8, then the height using 30-60-90 triangles is , **E.**
13. Shoelace theorem, remember to keep the order of the vertices in the correct order, **C.**
14. Using the distance formula between the vertices in the correct order, gives the four sides as and. Adding these up gives **C.**
15. A frustum is essentially a large cone that has a smaller cone cut off from its top. We solve this problem by finding the area of the original large cone and subtracting the area of the small cone taken off. Using similar triangles, we know that the small radius over the small cone’s height is equal to the big radius over the large cone’s height. Solving gives x = 6. So, the original large cone had a total height of 6 + 6 = 12. The volume of a cone is , so the big cone has volume . The small cone has radius 2 and height 6, so its volume is 8π. The volume of the frustum is , **B.**
16. The inverse of the contrapositive of the converse of a statement reverts itself back to its original form (does not change it), so we are left with finding the inverse of the statement: “If it is raining, then I will not visit the Great Wall of China.” The inverse of a statement negates the original statement so we are left with “If it is not raining, then I will visit the Great Wall of China.” **A**
17. If we were to roll out a cylinder to look at its “net,” we are left with 2 circles (the top and bottom circles of the cylinder), and a rectangle. First solving the area of the 2 circles, they both have a radius of 4 cm so the area of both of them is which is equal to The rectangle height is given as 6 mm, or 0.6 cm. The width is the circumference of one circle which is cm. Multiplying these two gives us the area of the rectangle which is . Adding and gives us the surface area which is . Looking at the answer choices, there is no . Thus, we convert it to , giving us , which is one of the answer choices, **D.**
18. Case 1: Set , so we have , which makes . We then have to plug it back into the equation, giving us , and , eliminating option A and D.

Case 2: , so we have , which makes . Plugging it back in, we get , and , eliminating option B.

This leaves to be the only angle measure that does not work, thus the correct answer is **C.**

1. Taking a close look at the units again, first we solve the volume in feet. 2 ft. 4 in is equal to feet, so the volume of the cube is , which is none of the answers. Then multiplying this by to convert it to , we get which is also none of the answers, leaving us with **E**.
2. Applying the distance formula, we get equaling to . This can not be further simplified so the answer is **D.**
3. The fastest route for Genghis Khan to get to Rahul will be a straight line, however, there is a river so this is impossible. Instead, we can reflect the Genghis Khan’s coordinate over the river, then calculate the distance from there and we will have the shortest. Reflecting (4,7) over the river at gives the coordinate (21,7). Now using the distance formula, the distance between (21,7) and (-1,-4) is , which is equal , giving us the answer; **D.**
4. Converting meters to millimeters first, 0.06 m = 60mm. Then, dividing the diameter in half gives us the radius which is 0.45 mm. Now, solving the volume of the cylinder using the formula , we get , which gives us a volume of ; **B**.
5. AAA, or better known as Angle-angle-angle, does not work because just because the angles are the same does not mean the sides are equal. ASA, SSS, and SAS congruence can prove that two triangles are congruent. **A**
6. V = 8, W = 0, X = 7, Y = 27, Z = 12. Using this, V \* X \* Z is equal to 8 \* 7 \* 12 = 672; **A.**
7. Before we start any calculation, we see that there is a “\* W” which will make the whole equation equal to 0, since W = 0. **E (0)**
8. Area of a circle is equal to , so . However, it is a semi-circle so we divide the area by 2 and get ; **A.**
9. Using Euler’s formula (Faces + Vertices –Edges = 2), we get the number of vertices as 12; **C**.
10. For solving the area of a cyclic quadrilateral, we can apply the Brahmagupta’s formula: In this formula, s is equal to the semi perimeter. a,b,c,d are each of the side lengths of the quadrilateral. Applying this formula, we get that s = 20, and then which is equal to , or when simplified, ; **B.**
11. In a circumscribed quadrilateral, the sum of opposite sides are equal. In other words, in circumscribed quadrilateral WXYZ, WX + YZ = XY + ZW. Applying this, we get 24 + 22 = 17 + ZW. So ZW = 29; **D.**
12. Ceva’s Theorem states that for any triangle such as the one in the problem, . Substituting in the measurements given in the problem, , so . Since the total length of is , and ;**C.**